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Domain Walls in a FRW Universe

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Abstract

We solve the equations of motion for a scalar field with domain wall boundary conditions in a Friedmann-Robertson-Walker (FRW) spacetime. We find (in agreement with Basu and Vilenkin) that no domain wall solutions exist in de Sitter spacetime for $h \equiv H/m \geq 1/2$, where H is the Hubble parameter and m is the scalar mass. In the general FRW case we develop a systematic perturbative expansion in h to arrive at an approximate solution to the field equations. We calculate the energy momentum tensor of the domain wall configuration, and show that the energy density can become *negative* at the core of the defect for some values of the non-minimal coupling parameter ξ . We develop a translationally invariant theory for fluctuations of the wall, obtain the effective Lagrangian for these fluctuations, and quantize them using the Bunch-Davies vacuum in the de Sitter case. Unlike previous analyses, we find that the fluctuations act as zero-mass (as opposed to tachyonic) modes. This allows us to calculate the distortion and the normal-normal correlators for the surface. The normal-normal correlator decreases logarithmically with the distance between points for large times and distances, indicating that the interface becomes rougher than in Minkowski spacetime.

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1 Introduction

The formation of topological defects in a cosmological phase transition is a basic result in particle cosmology lore [1, 2]. The existence of defects provides us with a powerful tool which both constrains particle theory models, such as those containing domain walls [3] and magnetic monopoles [4], and solves some outstanding cosmological problems, such as the formation of structure with the use of cosmic strings [2, 5]. Furthermore, during out-of-equilibrium phase transitions, topological defects such as interfaces (domain walls) are important ingredients for the dynamics of phase separation and phase coexistence.

It has always been assumed that if a field theory defined in Minkowski spacetime admits topological defects, the same will be true in an expanding universe, and that furthermore, there will not be any significant differences in the physical characteristics of these defects. Recently, however, Basu and Vilenkin [6] have shown that defects in a de Sitter background can have properties that are quite different from defects in flat spacetime. In this work, we continue and extend this program by analyzing the equations of motion for a domain wall in a background Friedmann-Robertson-Walker (FRW) spacetime. We find that although these equations cannot be solved analytically in general, a systematic perturbative expansion can be set up, with the small parameter $h \equiv H/m$ being the ratio of the correlation length of the field to the horizon size. Such a perturbative expansion arises naturally from the requirement that the width of the domain wall be much smaller than the particle horizon, to allow for kink boundary conditions within the microphysical horizon. The zeroth order solution is taken to be the standard flat spacetime domain wall configuration, except that it is a function of the *physical* spatial coordinate. The higher order terms in h are then solved for in a systematic perturbative fashion. We can also analyze the de Sitter case numerically; we find that we agree with the results of Basu and Vilenkin concerning the fact that for sufficiently large values of h , no wall solutions exist – when the horizon size is smaller than the domain wall width, a kink configuration cannot fit in the horizon.

We next perform an expansion in the physical spatial coordinate that is nonperturbative in h , and use it to examine the behavior of the energy momentum tensor for the wall near the origin. This is of interest due to the possibility that a more natural form of inflation may occur in the core of the defect, where the field is trapped in the unstable vacuum [7].

In curved spacetime, renormalization arguments suggest that, in general, a coupling to the Ricci scalar has to be introduced in the bare Lagrangian. We find that such coupling is responsible for very remarkable effects that result in a *change of sign* of the stress tensor at the origin. In fact, we see that this effect can also happen in flat spacetime, if the so-called *improved* stress tensor [8] is used.

We derive the effective Lagrangian for the fluctuations perpendicular to the wall. We find corrections beyond the “Nambu-Goto” action (the 3-volume swept by the worldsheet of

the wall) in a consistent expansion in derivatives. We also find non-trivial contributions to the surface tension from gravitational effects.

We use this effective Lagrangian to address the question of whether the shape of the wall fluctuates, or whether it remains flat at long distances and long times after the formation of the defect. Garriga and Vilenkin [9] have discussed this question to a certain extent, however we find that by using collective coordinates the long-wavelength fluctuations are identified with Goldstone modes and the instability found by them is avoided.

In the next section we set up our ansatz and perturbation expansion. Section 3 deals with the energy momentum tensor for the wall in an FRW universe. We find that interesting behavior can occur by making use of the coupling $\xi \mathcal{R} \phi^2$ of the field to the Ricci scalar \mathcal{R} . In section 4, we analyze fluctuations of the wall. We differ from Garriga and Vilenkin in our treatment of translational invariance. The fact that the wall is located at a particular point along the z -axis, say, would appear to break this symmetry. However, translational invariance can be restored by use of the collective coordinate method. Doing this, we arrive at a true Nambu-Goldstone mode for the fluctuations, as opposed to one with negative mass² as found by Garriga and Vilenkin. Several equal (comoving) time correlation functions that determine the behavior of the shape of the wall are computed, and we find that their spatial fall-off is much slower than in flat Minkowski spacetime at long times. This leads us to the conclusion that the surface remains “rough” at large separations.

Section 5 contains our concluding remarks and some cosmological implications of our results.

2 The Domain Wall

Domain walls (interfaces) are field configurations with non-trivial topological boundary conditions. These boundary conditions demand that the field vary substantially within a typical spatial scale that is usually determined by the (finite temperature) correlation length $\xi \approx m^{-1}$, with m the, in general temperature-dependent, effective mass of the field.

In FRW cosmologies there is another important scale, the microphysical horizon size $d_h = H^{-1}$ where H is the Hubble parameter. Causality implies that the region in which the scalar field can vary appreciably must be subhorizon sized, since only then can the boundary conditions that define the wall fit inside the horizon. Thus we expect that the notion of a domain wall will only make sense if $H/m \leq 1$.

When $H/m \ll 1$, we expect that the domain wall profile will exhibit only small deviations from the Minkowski spacetime profile. We can then study the differences in a power series expansion in $H/m = \xi/d_h$.

Consider a scalar field ϕ with potential

$$U = \frac{\lambda}{4}(\phi^2 - v^2)^2 + \frac{\xi}{2}\mathcal{R}\phi^2 \quad (1)$$

in a FRW spacetime with metric $g_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2)$, and Hubble parameter $H \equiv \dot{a}/a$ (where $\dot{a} \equiv da/dT$). The second term couples the field to the Ricci scalar $\mathcal{R} = 6(2H^2 + \dot{H})$. Such a term is required for consistency as it will be generated by quantum corrections in general; the only fixed point is $\xi = 1/6$ which corresponds to conformal coupling [10]. The equation of motion is

$$\frac{\partial^2 \phi}{\partial T^2} + 3H \frac{\partial \phi}{\partial T} - \frac{1}{a^2} \nabla^2 \phi + \frac{\partial U}{\partial \phi} = 0 \quad (2)$$

where ∇ is the derivative with respect to *co-moving* spatial coordinates $\{X, Y, Z\}$. It is convenient to define the following dimensionless quantities

$$\begin{aligned} m &= \sqrt{\lambda} v, & \{x, y, z, t\} &= m\{X, Y, Z, T\}, & h &= H/m, \\ \eta &= \phi/v, & \omega &= a(t)z, & \beta &= 4 + \dot{H}/H^2, & \mu^2 &= 1 - 6\xi h^2(\beta - 2). \end{aligned} \quad (3)$$

Note that ω is a (dimensionless) physical coordinate. For power-law expansion (PLE) $a(t) = t^n$, we have $\beta = 4 - 1/n$. In the radiation dominated (RD) and matter dominated (MD) flat universes respectively, $\beta_{RD} = 2$ and $\beta_{MD} = 2.5$. In de Sitter spacetime $\beta_{dS} = 4$ (or equivalently, we can take $n = \infty$).

We will assume a domain wall (kink) along the z -axis and that the corresponding field configuration is independent of the transverse coordinates, so that we are considering a flat interface.

The scale factor dividing the (comoving) Laplacian in eq. (2) suggests the following ansatz for the kink configuration:

$$\eta(z, t) = \eta(\omega, h(t)). \quad (4)$$

Such a solution obeys

$$(1 - h^2 \omega^2) \eta'' - \beta h^2 \omega \eta' + \mu^2 \eta - \eta^3 = \frac{1}{n^2} h^4 \frac{\partial^2 \eta}{\partial h^2} - \frac{2}{n} h^3 \omega \frac{\partial \eta'}{\partial h} + \left(\frac{2}{n^2} - \frac{3}{n} \right) h^3 \frac{\partial \eta}{\partial h} \quad (5)$$

where a prime means $\partial/\partial\omega$. The right-hand side (RHS) vanishes for de Sitter spacetime, and is $\mathcal{O}(h^4)$ for PLE.

In the case of de Sitter spacetime, the effect of non-zero conformal coupling ξ (*i.e.* $\mu \neq 1$) can be absorbed in a simple rescaling of variables:

$$\eta = \mu \bar{\eta}, \quad \omega = \bar{\omega}/\mu, \quad h = \mu \bar{h}. \quad (6)$$

For PLE, this introduces additional $\mathcal{O}(h^4)$ terms on the RHS of eq. (5), since $\dot{\mu} \neq 0$.

Except to discuss the energy-momentum tensor $T_{\mu\nu}$, we will hereafter work in the rescaled theory (and drop the bars), writing

$$(1 - h^2\omega^2)\eta'' - \beta h^2\omega\eta' + \eta - \eta^3 = \begin{cases} 0 & (\text{de Sitter}) \\ \mathcal{O}(h^4) & (\text{PLE}) \end{cases} \quad (7)$$

and taking “soliton boundary conditions” $\eta(\pm\infty, h) = \pm 1$. All results for PLE are thus valid only to $\mathcal{O}(h^2)$.

2.1 Walls in de Sitter Spacetime: Numerical Solution

The differential equation (5) is, in general, a partial differential equation in two variables, and not easily amenable to numerical study. As we see in eq. (7), ignoring the RHS would introduce errors at $\mathcal{O}(h^4)$ for PLE. However, in de Sitter spacetime, where h is constant, we have an ordinary differential equation, for which we can find exact numerical solutions. Indeed, this was done by Basu and Vilenkin [6], and our results appear to agree with theirs.

Our results are plotted as thin curves in Fig. 1. The slope at the origin was chosen (by a shooting algorithm) by requiring smoothness through the point $\omega = 1/h$ (marked with a tick). As found by Basu and Vilenkin [6], solutions only exist when $h^2 < 1/4$. (For PLE, if one insists on ignoring the RHS of eq. (7), solutions only exist when $h^2 < 1/\beta$.)

We see the kink has been flattened out by the effects of the cosmological expansion. The slope at the origin $a_1 \equiv \eta'(0, h)$ decreases with h^2 as shown in Fig. 2, vanishing at $h^2 = 1/4$. In other words, for a topologically stable solution to exist, the horizon size must be greater than the correlation length (by a factor of 2).

2.2 Expansion in h^2

Although a numerical solution is available in the case of de Sitter, the general case is extremely difficult to analyze numerically, since we now have a non-linear partial differential equation in two variables. However, when $h^2 \ll 1/\beta$, we can solve eq. (7) in a systematic perturbative expansion in h^2 , and the first-order result will be valid for both de Sitter and PLE cosmologies. We write

$$\eta(\omega, h) = \eta_s(\omega) + \sum_n h^{2n} \delta^{(n)}(\omega) . \quad (8)$$

Here η_s is the kink configuration which solves eq. (7) for $h = 0$ (flat spacetime):

$$\eta_s(\omega) = \tanh(\omega/\sqrt{2}) \quad (9)$$

This $h = 0$ kink is shown as a dashed curve in Fig. 1. Note that $\eta'_s(0) = 1/\sqrt{2}$.

Since η_s has the correct asymptotic behavior, we must set $\delta^{(n)}(\pm\infty) = 0$. To leading order in h^2 , $\delta(\omega)$ obeys

$$\hat{O}_s \delta^{(1)} = \frac{d\delta^{(1)}}{d\omega^2} + \delta^{(1)} - 3\eta_s^2(\omega)\delta^{(1)} = j(\omega) \equiv \eta_s''(\omega)\omega^2 + \beta\eta_s'(\omega)\omega. \quad (10)$$

The solution may be found by elementary methods once a solution of the homogeneous equation is found. Such a solution is easily available, since translational invariance of the unperturbed differential equation guarantees that

$$\delta_1(\omega) = \eta_s'(\omega) = \frac{1}{\sqrt{2}} \text{sech}^2\left(\frac{\omega}{\sqrt{2}}\right), \quad (11)$$

is a “zero mode” of the operator \hat{O}_s of quadratic fluctuations around the kink configuration.

Another linearly independent solution $\delta_2(\omega)$ having unit Wronskian with $\delta_1(\omega)$ can be found by elementary methods:

$$\delta_2(\omega) \equiv \frac{1}{4} \left[\sinh(\sqrt{2}\omega) + 3 \tanh\left(\frac{\omega}{\sqrt{2}}\right) + \frac{3\omega}{\sqrt{2}} \text{sech}^2\left(\frac{\omega}{\sqrt{2}}\right) \right], \quad (12)$$

where we have chosen this solution to vanish at the origin. Finally the first order solution can be written

$$\delta^{(1)}(\omega) = \delta_2(\omega) \int_0^\omega \delta_1(\zeta) j(\zeta) d\zeta - \delta_1(\omega) \int_0^\omega \delta_2(\zeta) j(\zeta) d\zeta + a\delta_1(\omega) + b\delta_2(\omega), \quad (13)$$

where the coefficients a, b have to be determined from the boundary conditions. Notice that $\delta_1(\omega)$ is symmetric whereas $j(\omega)$, $\delta_2(\omega)$ are antisymmetric functions of ω .

Note that $\delta_2(\omega)$ diverges exponentially as $\omega \rightarrow \pm\infty$. To render the solution finite and to satisfy the boundary conditions, we must choose:

$$b = - \int_0^\infty \delta_1(\zeta) j(\zeta) d\zeta. \quad (14)$$

The boundary condition at infinity *does not* determine the constant a . Notice that the term $a\delta_1(\omega)$ represents a local translation of the wall. Clearly the freedom of choice of a reflects the underlying translational invariance. If we demand that the wall position be at the origin, we must demand that $\delta^{(1)}(\omega = 0) = 0$; this leads to $a = 0$.

Thus the final solution to this order that satisfies the boundary conditions and keeps the wall centered at the origin is given by

$$\delta^{(1)}(\omega) = -\delta_2(\omega) \int_\omega^\infty \delta_1(\zeta) j(\zeta) d\zeta - \delta_1(\omega) \int_0^\omega \delta_2(\zeta) j(\zeta) d\zeta. \quad (15)$$

The results for de Sitter spacetime ($\beta = 4$) are shown as thick curves in Fig. 1. $\delta^{(1)}(\omega)$ for $\beta = 4$ is plotted in Fig. 3. This approximation underestimates the distortion away from

the $h = 0$ kink, and does not see the singularity at $h^2 = 1/4$, which is well beyond the regime of validity of the perturbative expansion in h^2 .

It is straightforward to see that the structure of the perturbative expansion persists to all orders. In fact we find that

$$\hat{O}_s \delta^{(n)} = j^{(n)} \quad (16)$$

with the same differential operator \hat{O}_s as in (10), and where the source term $j^{(n)}$ is obtained from the solutions up to order $n - 1$. Clearly the same functions $\delta_1(\omega)$, $\delta_2(\omega)$ generate the solutions, as these are the two linearly independent solutions of the homogeneous differential equation $\hat{O}_s \delta = 0$. Thus the general form of the solution is the same as (13) with $j^{(n)}$ replacing j and coefficients $a^{(n)}$, $b^{(n)}$ replacing a, b .

By the same arguments presented before, a finite solution at infinity requires that

$$b^{(n)} = - \int_0^\infty \delta_1(\zeta) j^{(n)}(\zeta) d\zeta. \quad (17)$$

The source term $j^{(n)}(\omega)$ is constructed from iterations up to order $(n - 1)$, and for $n = 1$ it is given by derivatives of the unperturbed kink solution that vanish at $\omega = \pm\infty$. It is straightforward to prove by induction that the source term $j^{(n)}(\omega)$ is antisymmetric and vanishes exponentially as $\omega \rightarrow \pm\infty$. Therefore $b^{(n)}$ is finite to all orders. Translational invariance suggests that $a^{(n)} = 0$ as before, and the the solution to order n is antisymmetric and this property ensures that the source term for the next order iteration is antisymmetric again. Thus we are led to the conclusion that the general form of the perturbative solution is given by eq. (15) with $j(\omega)$ replaced by $j^{(n)}(\omega)$, and the only complication in carrying out this program to any arbitrary order is to find the source term iteratively. Clearly this will be a highly non-local function because of the nested integrals, and the numerical evaluation will become more cumbersome in higher orders, but in principle this scheme will yield a consistent perturbative expansion. Clearly we have no way of determining the radius of convergence of such an expansion.

2.3 Expansion in ω

The field configuration corresponding to a domain wall has most of the gradient and potential energy difference (with respect to the broken symmetry vacuum) localized in a spatial region of the order of the correlation length. Most of the contribution to the energy momentum tensor, after subtracting off the vacuum value, will arise from this small region around the position of the domain wall. This motivates us to obtain the kink profile near the origin in a power-series expansion in the coordinate ω . This expansion is *nonperturbative* in h^2 .

The motivation for this expansion is the following: one would eventually like to study the full system of Einstein's equations and matter field in a self-consistent semiclassical manner.

In particular we have in mind the question of how the presence of domain walls affect the gravitational fields. Since for this task one only needs the behavior of the energy momentum tensor, and as we argued above most of the contribution to its components from a domain wall field configuration arise from a very localized region near the position of the interface, a power series expansion near the origin may capture most of the relevant physical features for an understanding of the back reaction of domain walls on the gravitational field. This argument becomes more relevant in view of the proposal of rapid inflation at the core of topological defects [7].

In the general case of de Sitter or power-law expansion we find the following behavior of the interface profile near the origin:

$$\eta(\omega, h) = a_1 \left[\omega + \frac{\beta h^2 - 1}{6} \omega^3 + \left(\frac{(\beta h^2 - 1)([6 + 3\beta]h^2 - 1)}{120} + \frac{a_1^2}{20} \right) \omega^5 + \mathcal{O}(\omega^7) \right] \quad (18)$$

The linear coefficient a_1 cannot be determined perturbatively, and will have to be found by solving the differential equation, its value being determined by requiring smoothness through $\omega = 1/h$. We notice, however, that as $h^2 \rightarrow 1/\beta$ from below, the first term dominates (even for large ω) and the profile flattens out to a straight line.

Since this expansion works best for h^2 close to $1/\beta$, but is correct only to $\mathcal{O}(h^2)$ for PLE, it is only useful in this case very close to the origin.

Given a value for a_1 (from the numerical solution) as input, we display this series [through $\mathcal{O}(\omega^5)$] for the de Sitter spacetime ($\beta = 4$) case as the thin dot-dashed curve in Fig. 1. For $h^2 \gtrsim 0.2$, where the solution is very flat, the small- ω expansion is valid out to well past $\omega = 1/h$. We will use these results in the following section.

3 The Energy-Momentum Tensor

We next turn to an examination of the energy momentum tensor for the domain wall profile in an FRW spacetime.

In general, the energy-momentum tensor of a scalar field configuration is given by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}, \quad \mathcal{L} = \frac{1}{2} g^{\alpha\beta} (\partial_\alpha \phi)(\partial_\beta \phi) - U(\phi) \quad (19)$$

where $g \equiv \det\{g_{\mu\nu}\}$, and we use

$$\frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2} \sqrt{-g} g_{\mu\nu} \quad (20)$$

At this stage we restore the non-minimal coupling to the Ricci scalar, since, as we argued above, such a term will be induced by renormalization if not present in the original

Lagrangian [10]. Its presence has two major effects: the first one is to modify the form of the energy momentum tensor, while the second is to modify the field configuration for the domain wall. Unlike the analysis of the equation of motion in the previous sections, for the case of the energy momentum tensor we can no longer rescale variables to get rid of ξ , since $(\delta\mathcal{R}/\delta g^{\mu\nu}) \neq 0$. Let U_0 be the potential for $\xi=0$, so $U = U_0 + (\xi/2)\mathcal{R}\phi^2$. Performing the variation of eq. (19), we find:

$$T_{\mu\nu} = (1 - 2\xi)\phi_{;\mu}\phi_{;\nu} + (2\xi - \tfrac{1}{2})g_{\mu\nu}g^{\alpha\beta}\phi_{;\alpha}\phi_{;\beta} + g_{\mu\nu}U_0 - \xi\phi^2(R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R) + 2\xi\phi(g_{\mu\nu}\square - \nabla_\mu\nabla_\nu)\phi \quad (21)$$

In our FRW metric, the non-zero components of $t_\mu^\nu \equiv T_\mu^\nu/(m^2v^2)$ are:

$$\begin{aligned} t_0^0 &= \frac{1}{2} [1 + (h\omega)^2] (\eta')^2 + \frac{1}{4}(\eta^2 - 1)^2 \\ &+ \xi \{ 3h^2\eta^2 + 6h^2\omega\eta\eta' - 2 [(\eta')^2 + \eta\eta''] \} \\ t_1^1 = t_2^2 &= \frac{1}{2} [1 - (h\omega)^2] (\eta')^2 + \frac{1}{4}(\eta^2 - 1)^2 \\ &+ \xi \{ (2\beta - 5)h^2\eta^2 + (2\beta - 2)h^2\omega\eta\eta' - 2 [1 - (h\omega)^2] [(\eta')^2 + \eta\eta''] \} \\ t_3^3 &= \frac{-1}{2} [1 + (h\omega)^2] (\eta')^2 + \frac{1}{4}(\eta^2 - 1)^2 \\ &+ \xi \{ (2\beta - 5)h^2\eta^2 + (2\beta - 2)h^2\omega\eta\eta' + 2(h\omega)^2 [(\eta')^2 + \eta\eta''] \} \\ t_3^0 = -a^2(t) t_0^3 &= a(t)h\omega \{ (\eta')^2 - 2\xi [(\eta')^2 + \eta\eta''] \} \end{aligned} \quad (22)$$

As usual, for PLE there are corrections at $\mathcal{O}(h^4)$.

The presence of the non-linear coupling to the Ricci scalar modifies the definition of the energy-momentum tensor. Even in the case of Minkowski spacetime, with $h=0$, this modification of the energy-momentum tensor produces some peculiar behavior in t_μ^ν near the origin, despite the fact that this coupling ξ does not affect the equations of motion. In this case, the wall solution is just η_s from eq. (9), and the non-zero components of t_μ^ν are:

$$t_0^0 = t_1^1 = t_2^2 = \left[\frac{1}{2} - 2\xi + \xi \cosh(\sqrt{2}\omega) \right] \text{sech}^4(\omega/\sqrt{2}) \quad (23)$$

For large enough ξ , the energy density is negative at the origin: $(t_0^0)(0) \leq 0$ for $\xi \geq 1/2$. For $\xi < 0$, a region of negative energy density occurs away from the origin.

We can substitute the small- ω expansion $\eta = a_1\omega + \mathcal{O}(\omega^3)$ (which is correct even for $\xi \neq 0$) into eq. (22) to find the behavior at the origin. The non-zero components of $t_\mu^\nu(0)$ are:

$$t_0^0(0) = t_1^1(0) = t_2^2(0) = \frac{1}{4} + \frac{a_1^2}{2} - 2a_1^2\xi, \quad t_3^3(0) = \frac{1}{4} - \frac{a_1^2}{2}, \quad (24)$$

and the energy density at the origin is negative for

$$\xi > \frac{1}{4} + \frac{1}{8a_1^2}. \quad (25)$$

These results are consistent with eq. (23), where $a_1^2 = \frac{1}{2}$ [see equation (9)].

In Fig. 4 we show exact numerical calculations of t_0^0 for de Sitter spacetime with $h^2 \neq 0$. The behavior of t_0^0 for small values of h^2 (such as $h^2 = 0.02$ in Fig. 4a) is qualitatively similar to eq. (23). For larger h^2 (such as $h^2 = 0.10$ in Fig. 4b) another effect enters, namely that the wall profile $\bar{\eta}(\bar{\omega})$ is really determined by $\bar{h} = h/\mu$, as defined in eq. (6). Wall solutions in de Sitter spacetime only exist for $\bar{h}^2 < 1/4$, which implies

$$h^2 < \frac{1}{4 + 12\xi} \quad (26)$$

For example, with $h^2 = 0.10$, the wall profile flattens away to $\eta = 0$ as $\xi \rightarrow 1/2$ (where $\bar{h}^2 \rightarrow 1/4$), and $t_0^0(\omega) = 1/4$.

The lesson that we learn from this analysis is that the coupling to the Ricci scalar can dramatically modify the behavior of the energy momentum tensor near the origin, and may be an interesting possibility for topological inflation at the core of defects [7]. These effects can even arise using the “improved” energy momentum tensor [8] in Minkowski spacetime, if one starts with a general curved spacetime and then takes the flat limit.

4 Fluctuations of the Wall

Up to this point our study has focused on the description of a “flat” interface or domain wall, that is the field profile varies only along the direction perpendicular to the domain wall but is constant on the perpendicular directions. However there will be fluctuations both quantum and thermal that will tend to distort locally the interface. An important question to address is the following: are these fluctuations “small” in the sense that the wall remains flat at long distances, or are the fluctuations important so that the wall becomes “rough”? We will assume that the system is at zero temperature and that only quantum fluctuations are important.

We begin by deriving the effective action for the fluctuations of the interface and then proceed to calculate relevant correlation functions.

We depart from the treatment of Garriga and Vilenkin [9], in that we sacrifice *explicit* covariance to treat translational invariance in terms of collective coordinates; however, the final result for the action will be fully covariant. This procedure, borrowed from the usual scheme to quantize the collective coordinates associated with translations of soliton solutions [11] has many advantages.

The position of the interface explicitly breaks translational invariance. However, a rigid translation of the interface should cost no energy due to the underlying translational symmetry. Thus the fluctuations perpendicular to the interface should be represented by *massless* degrees of freedom since, locally, they represent translations of the interface. These are the

capillary waves or Goldstone bosons [12, 13, 14] associated with the breakdown of the translation symmetry. Translational symmetry is then restored by quantizing these fluctuations as collective coordinates [11].

In terms of dimensionless comoving coordinates and the dimensionless field η , the action is (d^4x is the comoving volume element in dimensionless units)

$$I = \frac{1}{\lambda} \int d^4x a^3(t) \left\{ \frac{1}{2} \left(\frac{\partial \eta}{\partial t} \right)^2 - \frac{1}{2a^2(t)} (\nabla \eta)^2 - \frac{1}{4} (\eta^2 - 1)^2 \right\}. \quad (27)$$

We want to incorporate fluctuations of the interface solution (kink) and to obtain the effective action for the long-wavelength modes. In order to understand how to achieve this goal, it proves convenient to recall how this procedure works in Minkowski spacetime.

4.1 Goldstone Modes in Flat Spacetime

Consider first a static kink along the z -direction. As is well known [11] there is a zero mode of the linear fluctuation operator

$$\delta_0 \eta \approx \frac{\partial \eta_s(z - z_0)}{\partial z} \quad (28)$$

where z_0 is the position of the kink and η_s is the kink solution. This mode corresponds to a translation of the position of the kink, because $\eta_s(z - z_0) + \alpha \delta_0 \eta \approx \eta_s(z - z_0 + \alpha)$. Translational invariance guarantees that such a perturbation does not change the energy and thus is a zero mode of the linear fluctuation operator. Now consider a kink in three space dimensions, corresponding to a wall along the z -axis and centered at z_0 . A perturbation of the form

$$\delta \eta_p(x, y, z - z_0) = a_p \frac{\partial \eta_s(z - z_0)}{\partial z} e^{i\vec{p}_\perp \cdot \vec{x}_\perp}, \quad (29)$$

with $\vec{p}_\perp \cdot \vec{x}_\perp = p_x x + p_y y$, is an eigenmode of the linear fluctuation operator with eigenvalue $\omega_p = \sqrt{p_x^2 + p_y^2}$ [12]. These fluctuations of the interface correspond to the Goldstone bosons of the broken translational symmetry, and are the capillary waves of the interface [12, 13, 14]. The perturbed solution

$$\begin{aligned} \eta_s(z - z_0) + \sum_p \delta \eta_p(x, y, z - z_0) &\approx \eta_s(z - f(x, y)) \\ f(x, y) &= z_0 - \sum_p a_p e^{i\vec{p}_\perp \cdot \vec{x}_\perp} \end{aligned} \quad (30)$$

corresponds to a *local translation* of the interface. The a_p correspond to “flat directions” in function space. Just as in the one-dimensional case, these Goldstone modes cannot be treated in perturbation theory, because arbitrarily large a_p for $\vec{p}_\perp \rightarrow 0$ can be accessed at

no cost in energy. We will borrow results from the one-dimensional procedure and quantize these modes as “collective coordinates” [15].

Besides these Goldstone modes there are massive modes corresponding to the higher energy states of the one-dimensional kink with dispersion relation $E(p_\perp) = \sqrt{p_\perp^2 + 3m^2}$ [11, 12]. Because of this gap in the energy spectrum, we can safely concentrate on the long-wavelength fluctuations of the interface and obtain an effective action for these Goldstone modes, treated as collective coordinates. The coordinates a_p or the field $f(x, y)$ are now fully quantized. In a path integral quantization procedure, collective coordinate quantization amounts to a functional integral over all configurations of $f(x, y)$. Clearly this procedure restores translational invariance because now the field is invariant under $z \rightarrow z + \delta$, $f(x, y) \rightarrow f(x, y) + \delta$ and the field $f(x, y)$ is now functionally integrated with a translational invariant measure. Passing from the original integration variables in the functional integral to the new variables including $f(x, y)$ involves a Jacobian which is seen to be unity to the order that we are working in (see below). For a thorough exposition of collective coordinate quantization in the path integral and Hamiltonian forms, the reader is referred to the original literature [11, 15, 16, 17].

4.2 Goldstone Modes in a FRW Cosmology

After this digression in Minkowski spacetime we are ready to extend these observations to a FRW cosmology. Let $\eta_s(\omega)$ be a solution to eq. (7) (the h -dependence will now be implicit), and let us take the small-gradient limit, in which

$$f_{xx}, f_{yy}, f_{tt} \ll 1; \quad hf_x, hf_y, hf_t \ll 1 \quad (31)$$

($f_{xx} \dots$ are second derivatives with respect to the dimensionless comoving coordinate x , *etc.*) This approximation is consistent with our purpose of studying the long-wavelength fluctuations on distances such that $1 \ll x, y \ll h^{-1}$. We look for a profile of the form

$$\eta(z, f(x, y, t), t) = \eta_s(\xi(z, x, y, t)) \quad (32)$$

Note that the substitution $\xi(z, x, y, t) = a(t)[z - f(x, y, t)]$ does *not* in general solve the equations of motion, but

$$\xi(z, x, y, t) = a(t) \frac{[z - f(x, y, t)]}{\sqrt{1 + f_x^2 + f_y^2 - a^2(t)f_t^2}} \quad (33)$$

does, in our small-gradient approximation.

The denominator in eq. (33) has an important physical meaning. The function $f(x, y, t)$ determines the position of the domain wall (interface). This function induces a metric $g_{ab}^{(3)}$

on the $2 + 1$ dimensional world-volume swept out by the wall, and we find, in terms of dimensionless comoving variables,

$$\sqrt{g^{(3)}} = a^2(t) \sqrt{1 + f_x^2 + f_y^2 - a^2(t) f_t^2}. \quad (34)$$

The effective action for the displacement field $f(x, y, t)$ in the long-wavelength approximation, is found by following the usual procedure [12, 13, 14] which consists of computing the action for the profile (32) with (33) (this is also identified with the effective action for the collective coordinate [15]). After integrating by parts and discarding surface terms, we find

$$\begin{aligned} I = & -\frac{1}{\lambda} \int d^3x a^2(t) \left[C_0 - C_2 \left(5 + 2 \frac{\hbar}{h^2} \right) \frac{h^2}{2} \right] \sqrt{1 + f_x^2 + f_y^2 - a^2(t) f_t^2} \\ & + \frac{C_2}{8\lambda} \int d^3x \frac{a(t)^2}{(1 + f_x^2 + f_y^2 - a^2(t) f_t^2)^{3/2}} \\ & \left\{ \left[\partial_t (1 + f_x^2 + f_y^2 - a^2(t) f_t^2) \right]^2 - \right. \\ & \left. \frac{\left[\partial_x (1 + f_x^2 + f_y^2 - a^2(t) f_t^2) \right]^2}{a(t)^2} - \frac{\left[\partial_y (1 + f_x^2 + f_y^2 - a^2(t) f_t^2) \right]^2}{a(t)^2} \right\}, \end{aligned} \quad (35)$$

where d^3x represents the $2 + 1$ dimensional comoving volume element, and we have used $\eta_s(-\xi) = -\eta_s(\xi)$ to eliminate the integrations linear in ξ as well as having defined:

$$\begin{aligned} C_0 &= \int d\xi \left[\frac{1}{2} \left(\frac{\partial \eta_s}{\partial \xi} \right)^2 + \frac{1}{4} (\eta_s^2 - 1)^2 \right] \\ C_2 &= \int d\xi \left(\frac{\partial \eta_s}{\partial \xi} \right)^2 \xi^2. \end{aligned}$$

This is the final form of the action for the fluctuations of the interface. It exhibits the translational symmetry explicitly, and only contains derivative terms as is required of a Goldstone field. C_0 is identified with the (usual) flat spacetime surface tension, and we see that curved spacetime effects induce a renormalization of this surface tension. The first term (proportional to $\sqrt{g^{(3)}}$) is recognized as the equivalent to the “Nambu-Goto” action, which is essentially the total “world-volume” associated with the fluctuation field. The second term is thus a correction to the “Nambu-Goto” action; further corrections can be obtained in a systematic expansion in derivatives for the solution of the equations of motion.

Expanding I to quadratic order in the fluctuation field shows that the fluctuations are *massless*:

$$I_{\text{quad}} = I_0 + \frac{1}{\lambda} \int d^3x a^4(t) \left[C_0 - C_2 \left(5 + 2 \frac{\hbar}{h^2} \right) \frac{h^2}{2} \right] \left[\frac{1}{2} f_t^2 - \frac{1}{2a(t)^2} (f_x^2 + f_y^2) \right], \quad (36)$$

in contrast with the results found in reference [9].

An important quantity that gives information about the behavior of the fluctuations of the interface is the vector normal to the interface:

$$n^\mu = \frac{(-a^2(t)f_t, f_x, f_y, 1)}{a(t)\sqrt{1 + f_x^2 + f_y^2 - a^2(t)f_t^2}} \quad (37)$$

Its correlation functions, to be computed below, will give information on whether departures from a flat interface are significant.

At this point we would like to compare our result with those of Garriga and Vilenkin. These authors found that the fluctuations perpendicular to the interface are associated with instabilities that manifest themselves as a tachyonic mass for these fluctuations. From the discussion above, we are led to conjecture that the appearance of this tachyonic mass is the result of a quantization that does *not* preserve translational invariance.

Because our procedure *does* preserve translational invariance but is non-covariant in the intermediate steps, a direct comparison of our results is somewhat subtle. However, we can gain some insight by trying different parametrizations of our kink-profile. Consider the following the parametrization

$$\eta(z, f(x, y, t), t) = \eta_s(\chi(z, x, y, t)) \quad (38)$$

$$\chi(z, x, y, t) = \frac{a(t)z - F(x, y, t)}{\sqrt{1 + a^{-2}(F_x^2 + F_y^2) - F_t^2}} \quad (39)$$

instead of that of equations (32,33).

We see that χ is *not* invariant under the rigid translation $F \rightarrow F + F_0$, $z \rightarrow z + F_0$; instead a translation of the interface is compensated by a *time dependent* transformation $F \rightarrow F + F_0/a(t)$, $z \rightarrow F + f_0$. Because of the time derivative terms in the original action such a transformation is not an invariance of the action, which is then changed by terms proportional to \hbar (time derivatives of the scale factor), thus breaking translational invariance. Following the same steps leading to the effective action found above, neglecting higher derivative terms, integrating by parts and rearranging terms we arrive at:

$$\begin{aligned} I = & -\frac{1}{\lambda} \int d^3x a^2(t)(C_0 - C_2 h^2/2) \sqrt{1 + a^{-2}(F_x^2 + F_y^2) - F_t^2} \\ & + \frac{C_0}{2\lambda} \int d^3x F^2 \frac{a^2 h^2 (3 + \dot{h}/h^2)}{\sqrt{1 + a^{-2}(F_x^2 + F_y^2) - F_t^2}} + \mathcal{O}(F^2 F_i F_{jk}). \end{aligned} \quad (40)$$

Keeping only the quadratic terms in the action we find that the fluctuation field acquires a tachyonic mass, $m_F^2 = -3h^2$. This is the same value of the mass obtained by Garriga and Vilenkin [9]. Thus we conjecture that the scalar field that measures departures from a flat interface introduced by Garriga and Vilenkin is equivalent (at least to lowest order in

derivatives) to the scalar field F parametrizing the fluctuations of the interface as in equations (38,39). As explained above, this parametrization explicitly breaks rigid translational invariance in any FRW cosmology. The appearance of the mass term is understood as a consequence of this explicit breakdown of translational invariance, although this fact does not explain the tachyonic nature of the mass.

Thus it seems to us that there are advantages and disadvantages in both formulations. Whereas the formulation of Garriga and Vilenkin is desirable in that it maintains explicit covariance, there is the feature of instabilities associated with the tachyonic mass of the fluctuations, which if our analysis is correct, indicates the breakdown of translational invariance in the quantization procedure. On the other hand, our formulation, in terms of collective coordinate quantization, sacrifices explicit covariance, although the final result is covariant, but explicitly treats translational invariance and its restoration via the collective coordinate quantization. The collective coordinates represent massless fields as a consequence of this translational invariance.

4.3 Quantization of the Fluctuations

Quantizing the fluctuations of the interface allows us to answer some relevant questions about the dynamics of the interface. In particular we can answer the question that we posed at the beginning of the section that is whether the interface (wall) is flat or strongly fluctuating at long distances. In order to answer this question we must compute the correlation function of the vectors normal to the interface at long distances $1 \ll r \leq h^{-1}$ with r the distance on the two-dimensional surface of the interface.

Since we are interested in long distance physics, we will only consider slowly varying fluctuations of the fluctuation field f and neglect higher derivative terms in the action, keeping only the quadratic terms (one can be brave and pursue a perturbative expansion but we will content ourselves here with a lowest order calculation) in the action. Repeating eq. (36),

$$I \simeq -\frac{1}{2\lambda} \int d^3x a^2(t) \left[C_0 - C_2(5 + 2\dot{h}/h^2)h^2/2 \right] \left[f_x^2 + f_y^2 - a^2(t)f_t^2 \right] \quad (41)$$

The equations of motion for the field f are given by:

$$f_{tt} + f_t \left\{ 4h - \frac{C_2(5h\dot{h} + \ddot{h})}{[C_0 - C_2(5 + 2\dot{h}/h^2)h^2/2]} \right\} - \frac{\Delta f}{a^2(t)} = 0 \quad (42)$$

with Δ being the two-dimensional Laplacian.

In the most general case, the time dependence of the above equation is far too complicated to pursue analytically and one would have to resort to numerical integrations. Thus we

concentrate on the case of de Sitter expansion with scale factor $a(t) = e^{ht}$ that allows an analytic treatment.

The fluctuation field is expanded in terms of creation and annihilation operators and the mode functions which are solutions of the above equation of motion:

$$f(\vec{x}, t) = \frac{1}{\sqrt{\sigma A}} \sum_p \left[a_p e^{i\vec{p}\vec{x}} v_p(t) + a_p^\dagger e^{-i\vec{p}\vec{x}} v_p^*(t) \right] \quad (43)$$

$$\sigma = \frac{1}{\lambda} \left[C_0 - 5C_2 \frac{h^2}{2} \right] \quad (44)$$

where the “surface tension” σ has been absorbed in the definition of f to make it canonical and A is the (comoving) area of the (planar) wall.

Performing the change of variables on the mode functions $v_p = e^{-2ht} \chi_p(t)$ the equation for χ_p reads:

$$\frac{\partial^2 \chi_p}{\partial t^2} + \left(\frac{\vec{p}^2}{a^2(t)} - 4h^2 \right) \chi_p = 0. \quad (45)$$

whose solutions are linear combinations of the Bessel functions. The mode functions v_p are

$$v_p(t) = \sqrt{\frac{\pi}{4h}} e^{-2ht} \left[A_p H_2^{(1)} \left(\frac{pe^{-ht}}{h} \right) + B_p H_2^{(2)} \left(\frac{pe^{-ht}}{h} \right) \right]. \quad (46)$$

where the coefficients A_p, B_p are arbitrary so far.

Imposing canonical commutation relations between f and its canonical conjugate momentum $\Pi_f = \delta\mathcal{L}/\delta\dot{f}_t = \sigma f_t a^4$ leads to the relation

$$|A_p^2| - |B_p^2| = 1. \quad (47)$$

It is a well known feature of quantization in curved spacetimes [18] that a choice of A_p corresponds to a choice of vacuum state. Although we do not have a physical criterion to pick a particular vacuum state, we will choose the Bunch-Davies [19] vacuum for simplicity. Such a choice implies

$$B_p = 0. \quad (48)$$

Without loss of generality we can take $A_p = 1$. Finally, the field f is expanded in creation and annihilation operators with respect to the Bunch-Davies vacuum state as:

$$f(x, y, t) = e^{-2ht} \sqrt{\frac{\pi}{4h\sigma A}} \sum_p \left[a_p e^{i\vec{p}_\perp \cdot \vec{x}_\perp} H_2^{(1)} \left(\frac{pe^{-ht}}{h} \right) + a_p^\dagger e^{-i\vec{p}_\perp \cdot \vec{x}_\perp} H_2^{(2)} \left(\frac{pe^{-ht}}{h} \right) \right] \quad (49)$$

As mentioned above, we are interested on the long-wavelength fluctuations of the interface at long distances and at long times after its formation. Thus we will study the regime $t \gg h^{-1}$; $hr \leq 1$. Furthermore there are physical cutoffs that we must introduce: the (comoving) wavelengths cannot be bigger than the horizon, and because the nature of our

approximation cannot be shorter than the correlation length (the expansion is in derivatives, thus valid for slowly varying fields on the scale of the correlation length). Therefore integrals over wavevectors will be restricted to the interval $h \leq p \leq 1$.

The fluctuation field $f(x, y, t)$ measures the departure from a flat interface. Thus a quantity of interest is the correlation function of the vector normal to the space-like interface. This normal vector is obtained from the induced metric *on the two-dimensional surface* and given by

$$\vec{n} = \frac{(-f_x, -f_y, 1)}{\sqrt{1 + f_x^2 + f_y^2}} \simeq (-f_x, -f_y, 1 - f_x^2/2 - f_y^2/2). \quad (50)$$

The equal-time two-point correlation function is given by

$$\langle \vec{n}(\vec{x}, t) \cdot \vec{n}(\vec{y}, t) \rangle \simeq 1 + \frac{\pi^2 e^{-4ht}}{2h\sigma} \int_h^1 dp p^3 [J_0(pr) - 1] \left| H_2^{(1)} \left(\frac{pe^{-ht}}{h} \right) \right|^2. \quad (51)$$

notice from eq. (50) that $\langle \vec{n}(\vec{x}, t) \cdot \vec{n}(\vec{x}, t) \rangle = 1$ (to the order considered) and that the correlation function does not require any short distance subtraction. Its long time, long distance behavior is found to be

$$\langle \vec{n}(\vec{x}, t) \cdot \vec{n}(\vec{y}, t) \rangle \approx 1 - \frac{8\pi^2 h^3}{\sigma} \ln r/2 + \dots \quad (52)$$

where \dots stand for sub-leading terms that fall off fast at large r .

It is instructive to compare this result with that in Minkowski spacetime, for which $\langle \vec{n} \cdot \vec{n} \rangle \approx 1 - r^{-3}$. Clearly the interface is “rougher” in de Sitter spacetime. The result (52) raises the very interesting possibility of anomalous exponents in the correlation function. Notice that for large r the logarithmic (infrared) singularities become strong and eventually would have to be resummed in order to obtain meaningful long distance behavior. *Assuming* such a resummation of the lowest order result we obtain the long distance behavior:

$$\langle \vec{n}(\vec{x}, t) \cdot \vec{n}(\vec{y}, t) \rangle \approx r^{-\alpha} \quad (53)$$

$$\alpha = \frac{8\pi^2 h^3}{\sigma}. \quad (54)$$

This result has interesting implications. In particular, we find that at long times and distances the vectors normal to the interface are uncorrelated and the interface is “rough” rather than approximately flat with small fluctuations that fall-off rapidly at long distances.

This situation is very similar to that of the X-Y model and other models in statistical mechanics in $1 + 1$ Euclidean dimensions (typically massless scalar field theories) in which logarithmic singularities sum up to anomalous dimensions [20] much in the same way. This would indeed be a tantalizing possibility that needs to be studied further, perhaps by keeping higher order terms in the effective action and resumming using renormalization group arguments. This is certainly beyond the scope of this article.

Another quantity of interest that measures properties of the interface is the distortion [9]:

$$D^2(\vec{x}, \vec{y}; t) = \langle [f(\vec{x}, t) - f(\vec{y}, t)]^2 \rangle = 2 \langle f^2(\vec{x}, t) \rangle - 2 \langle f(\vec{x}, t) f(\vec{y}, t) \rangle \quad (55)$$

We evaluated this correlation function for large times and distances ($t \gg 1/h$; $r \gg 1 \gg hr$) and found

$$D^2(\vec{X}, \vec{Y}) \approx \frac{4\pi^2 h^3}{\sigma} r^2 \ln r. \quad (56)$$

This result is consistent with that of the normal-normal correlation function (52).

This correlation function shows once again that the interface is strongly fluctuating at long distances and cannot be considered flat.

5 Conclusions

In this article we have focused on the properties of domain walls in general FRW Universes. These defects will appear during phase transitions in typical inflaton theories and ultimately drive the dynamics of the process of phase separation. They may also contribute to density fluctuations and perhaps to structure formation.

Our results present a rich picture of the properties of these defects. After analyzing numerically the case of de Sitter spacetime and assuming that the boundary conditions on the field configuration require that the horizon size be larger than the typical correlation length we set up a systematic perturbative expansion in powers of $h = H/m$ and worked out in detail the first order correction from curved spacetime effects. We see then that even in situations in which the Universe is expanding slowly enough to allow us to set up our perturbation theory for defects, new phenomena occurs with definite cosmological implications.

In particular, we argue that in a general FRW spacetime, consistency (renormalizability) of the theory requires a coupling to the Ricci scalar. With this coupling the energy momentum tensor of defects can show some remarkable new features, including a negative energy density at the origin. Clearly this observation brings interesting possibilities for topological inflation near the center of the defect that must be studied within the full set of Einstein's equations in the presence of this configuration. For some values of the coupling to the Ricci scalar, we find that the energy density at the center is *lower* than at a distance of the order of the correlation length away from the center.

Such a behavior of the energy momentum tensor may provide an interesting mechanism for scalar density perturbations.

Using collective coordinate quantization of the fluctuations perpendicular to the wall, we obtained the effective Lagrangian for these fluctuations in the long-wavelength approxima-

tion. There are several noteworthy features of this effective Lagrangian: curved spacetime effects renormalize the surface tension, and we find systematic corrections to the “Nambu” action.

We have also shown how to quantize the fluctuations about a completely flat wall, and how the collective coordinate procedure yields the expected Goldstone mode associated with translational invariance. This analysis then allows us to compute correlation functions and to see that, at least in the de Sitter case, fluctuations are fairly strong at long distances and long times after formation of the defect leading to conclusion that the surface tends to be rougher than in flat spacetime. These results and the method developed for the quantization of the fluctuations may yield some insight on novel mechanisms to study density perturbations.

What we have done for walls can also be done for other defects, such as cosmic strings. These should exhibit some interesting behavior, especially for the fluctuations, since there will now be collective coordinates associated with the internal $U(1)$ symmetry whose spontaneous breaking gives rise to the string. Interesting possibilities are corrections to the string tension from curved spacetime effects and also corrections to the “Nambu-Goto” action from higher derivative terms, and perhaps novel behavior of the energy momentum tensor at the core of the defect when the coupling to the Ricci scalar is introduced. We are currently studying these and other issues.

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References

- [1] T.W.B. Kibble, J. Phys. **A9**:1387 (1976).
- [2] A. Vilenkin, Phys. Rep. **121**:263 (1985).
- [3] Ya. B. Zel'dovich, I. Yu. Kobzarev and L.B. Okun, Zh. Eksp. Teopr. Fiz. **67**:3 (1974) (Sov. Phys. JETP **40**:1 (1975)).
- [4] Ya. B. Zel'dovich and M. Yu. Khlopov, Phys. Lett. **79B**:239 (1978);
J.P. Preskill, Phys. Rev. Lett. **43**:1365 (1979).
- [5] M. B. Hindmarsh and T.W.B. Kibble, "Cosmic Strings", Preprint SUSX-TP-94-74, IMPERIAL/TP/94-95/5, NI94025, Los Alamos Archive hep-th/9411342 (1994).
- [6] R. Basu & A. Vilenkin, Phys. Rev. **D50**:7150 (1994).
- [7] A.D. Linde, Phys. Lett. **327B**:208 (1994);
A. Vilenkin, Phys. Rev. Lett. **72**:3137 (1994);
A.D. Linde & D.A. Linde, Phys. Rev. **D50**:2456 (1994).
- [8] R. Jackiw in "Current Algebra and Its Applications", Princeton University Press, Princeton, NJ (1972).
- [9] J. Garriga, A. Vilenkin, Phys. Rev. **D44**:1007; *ibid* **D45**:3469.
- [10] D. Boyanovsky, H. J. de Vega & R. Holman, Phys. Rev. **D49**:2769 (1994).
- [11] R. Rajaraman, "Solitons and Instantons" (North Holland 1984).
- [12] D. Jasnow in "Phase Transitions and Critical Phenomena" ed. Domb and Green (Academic Press-London) vol. 10, p. 269 (1986), and Rep. Prog. Phys. **47**:1059 (1984).

- [13] S. A. Safran, “Statistical Thermodynamics of surfaces, interfaces and membranes” (Frontiers in Physics, Addison Wesley, 1994).
- [14] D. J. Wallace, “Perturbative approach to surface fluctuations” Wallace, in Les Houches, Session XXXIX, 1982 - Recent Advances in Field Theory and Statistical Mechanics; J.-B. Zuber and R. Stora, eds. (Elsevier Science Publishers) pp. 173-216 (1984).
- [15] G.-L. Gervais and B. Sakita, Phys. Rev. **D11**:2943 (1975);
 Phys. Rev. **D16**:3507 (1977);
 Nucl. Phys. **B110**:93 (1976); *ibid* 113.
- [16] N. H. Christ and T. D. Lee, Phys. Rev. **D12**:1606 (1975).
- [17] E. Tomboulis, Phys. Rev. **D12**:1678.
- [18] N. D. Birrell and P.C.W. Davies “Quantum Fields in Curved Space”, Cambridge Univ. Press, 1982.
- [19] T. S. Bunch and P.C.W. Davies, Proc. R. Soc. London **360**:117 (1978).
- [20] A. M. Polyakov; “Gauge Fields and Strings” (Harwood Academic) (1987).

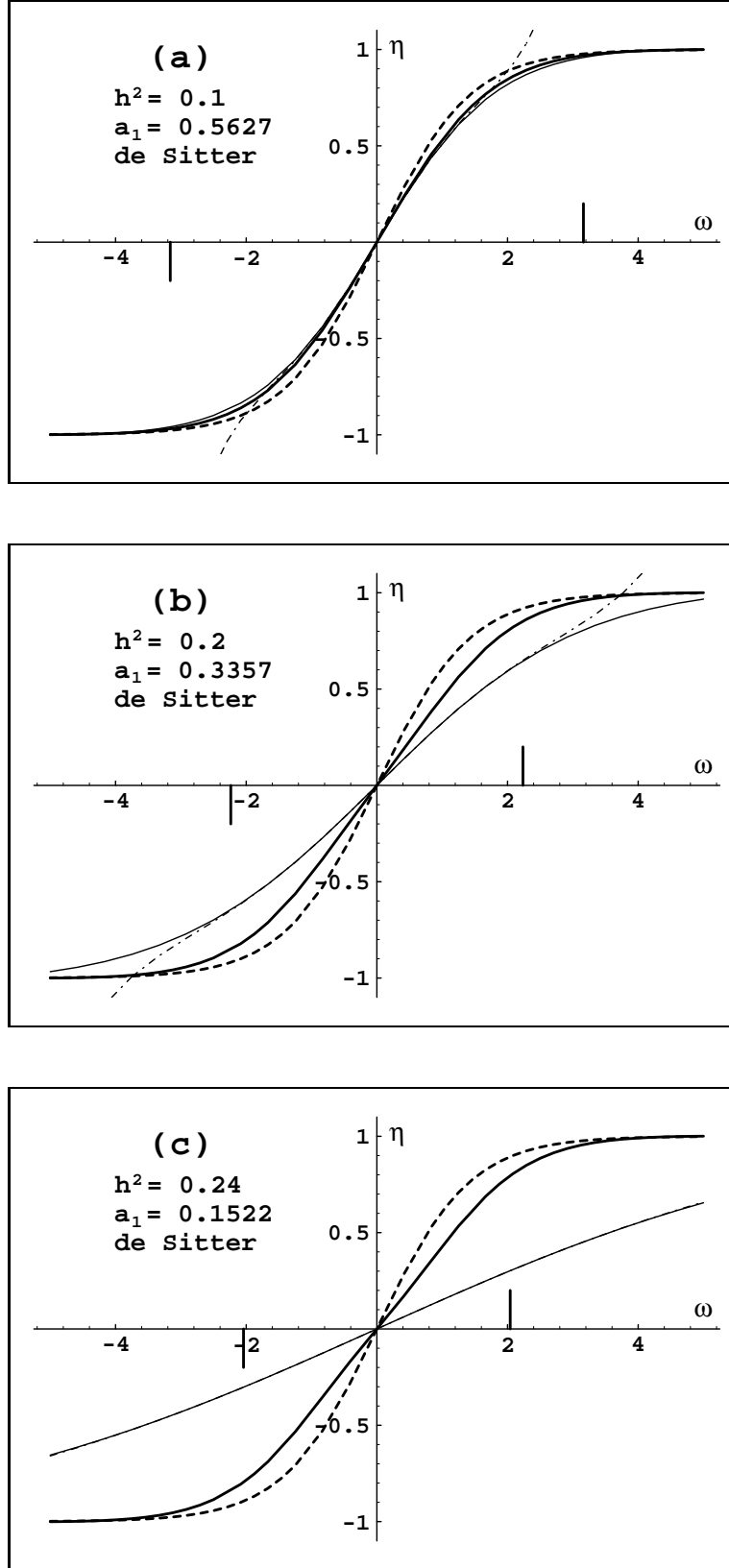


Fig. 1: Domain wall profiles for de Sitter space, $h^2 = \{.1, .2, .24\}$. Dash=kink, thin=exact, thick=small- h^2 , dot-dash=small- ω . $\omega = \pm 1/h$ is marked by vertical ticks.

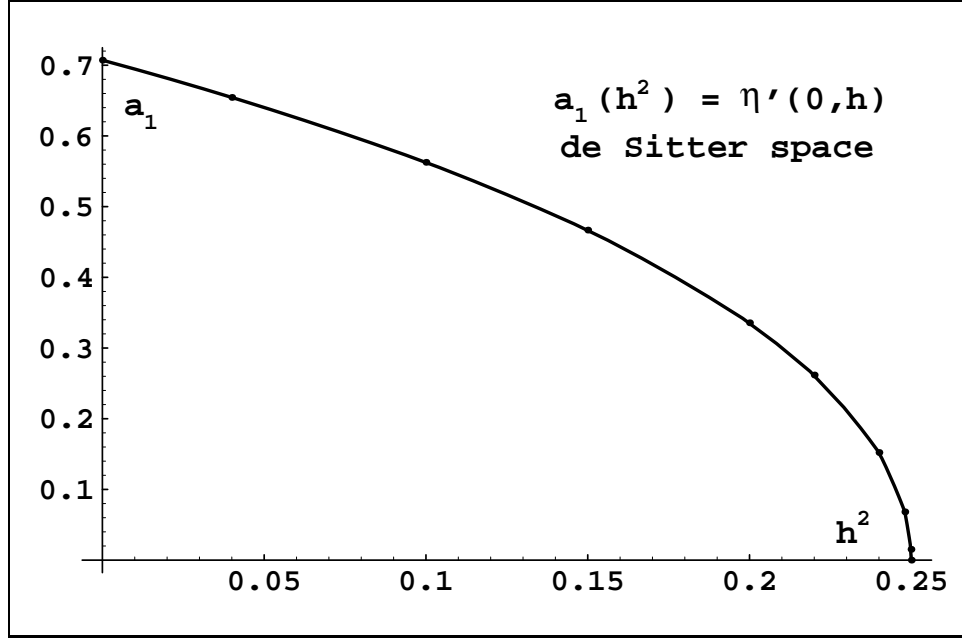


Fig. 2: $a_1 \equiv \eta'(0, h)$ vs. h^2 for de Sitter space.

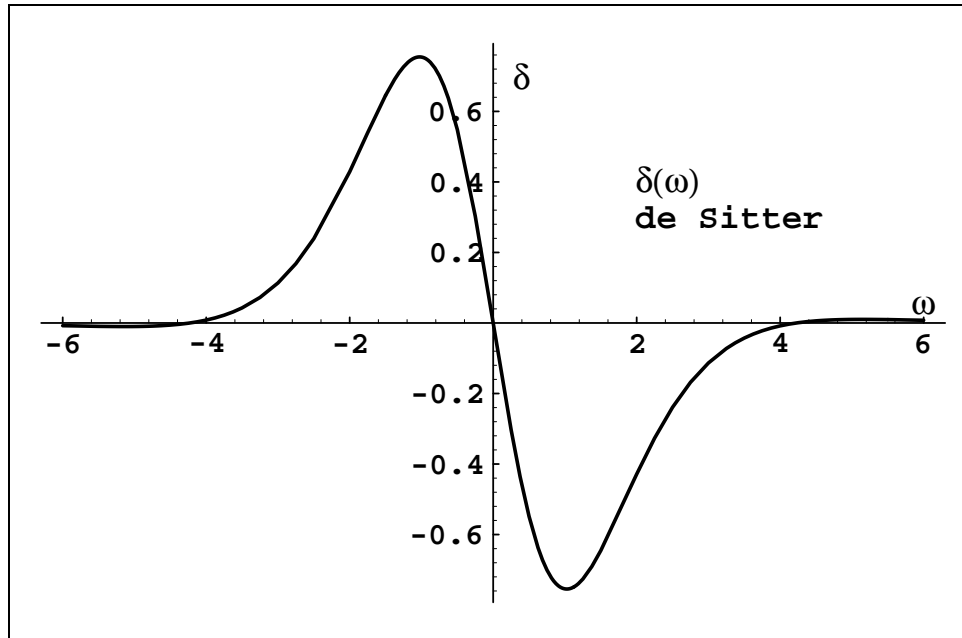


Fig. 3: $\delta(\omega)$ for de Sitter space.

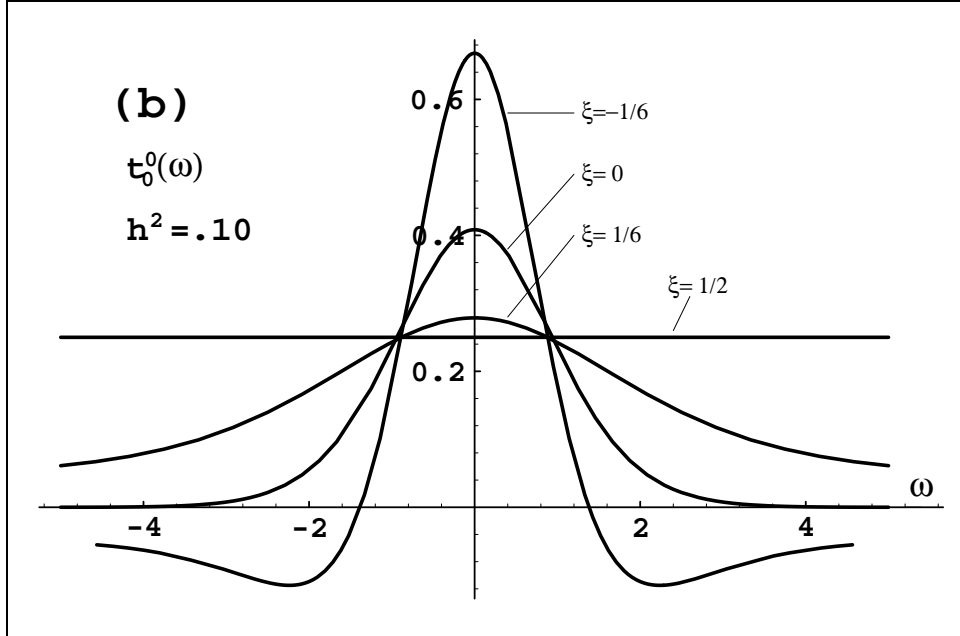
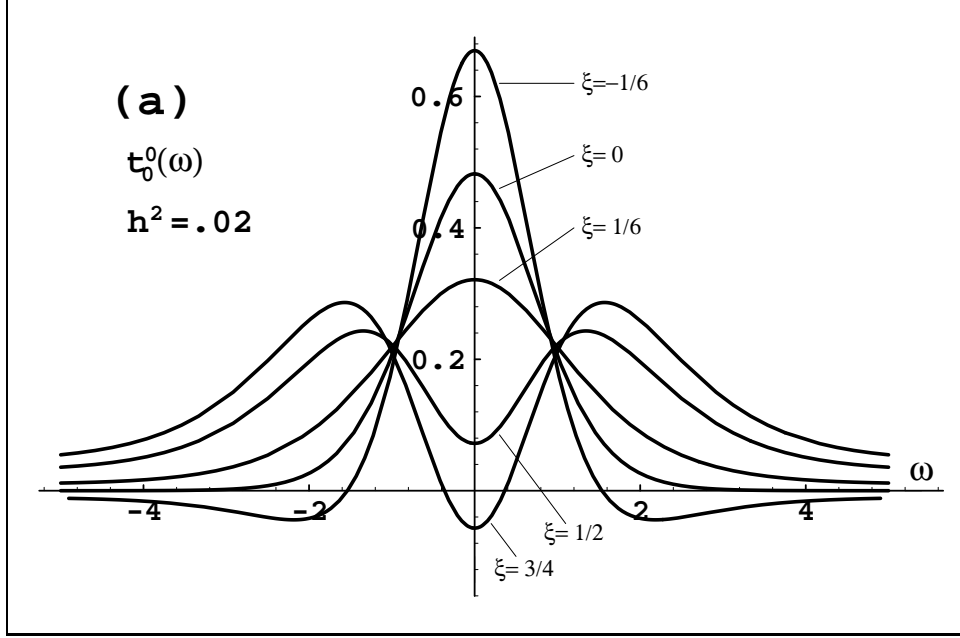


Fig. 4: $t_0^0(\omega)$ in de Sitter space, various ξ , $h^2 = \{.02, .1\}$.